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Global shape versus local feature: An angle illusion

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ABSTRACT

We have shown previously that the precision of angle judgments depends strongly on the global stimulus configuration: discrimination thresholds for angles that form part of isosceles triangles are up to 3 times lower than for those that form part of scalene triangles [Kennedy, G. J., Orbach, H. S., & Loffler, G. (2006). Effects of global shape on angle discrimination. *Vision Research*, *46*(8–9), 1530–1539]. Here, we investigated whether or not the perceived *size* of an angle (accuracy) is also affected by the overall shape of which it forms a part. Observers compared the relative sizes of angles contained in isosceles triangles with those of angles in scalene triangles and points of subjective equality were determined. For a reference angle of 60°, angles embedded in isosceles triangles were judged to be on average 14° larger than angles embedded in scalene triangles. This result is largely independent of the reference angle, triangle orientation and triangle size. Moreover, the effect is present whether or not triangles of different shapes enclose the same area, whether or not the side of the triangle opposite the angle is present and whether the triangle is outlined or defined by dots at its vertexes. In sum, our results provide evidence for a novel illusion where an angle embedded in an isosceles triangle is judged substantially larger than the same angle embedded in a scalene triangle. This finding demonstrates that mechanisms for computing angles are sensitive to the context within which angles are presented.

1. Introduction

It is widely accepted that object shape is processed in an hierarchical manner, starting at the earliest cortical level (primary visual cortex, V1; Hubel & Wiesel, 1968) and continuing in the inferior temporal region (Ungerleider & Mishkin, 1982; Van Essen & Gallant, 1994) with intermediate processing taking place in areas V2 (Hegdé & Van Essen, 2000; Ito & Komatsu, 2004; Kobatake & Tanaka, 1994) and V4 (Gallant, Connor, Rakshit, Lewis, & Van Essen, 1996; Pasupathy & Connor, 1999). This anatomical hierarchy corresponds to the increasing complexity of shapes for which neurons in these areas are selective, ranging from local line orientation in V1 (De Valois & De Valois, 1990; Hubel & Wiesel, 1962; Hubel & Wiesel, 1968) to highly complex objects such as faces in IT (Desimone, Albright, Gross, & Bruce, 1984; Perrett, Rolls, & Caan, 1982).

Psychophysical investigations into the characteristics of early filters in humans have shown these to be selective for spatial frequency and orientation (Campbell & Robson, 1968; Graham & Nachmias, 1971). Masking experiments have provided a characterisation of these early "orientation detectors", and this has been successfully compared with physiological data from primary visual cortex of cat and monkey (Wilson, 1991). Moreover, human orientation discrimination can be predicted by mechanisms that combine and compare the responses of several of these filters (Regan, 1982; Regan & Beverley, 1985).

After a single line, an angle comprising two lines could be considered a next step in a processing hierarchy for increasing shape complexity, given that angles could easily be extracted from images by combining the outputs of two mechanisms for orientation detection. It is therefore not surprising that angles have been widely proposed as important intermediate shape primitives (Attneave, 1954; Biederman, 1987), which can be used as building blocks for constructing more complex representations (Attneave, 1954; Pasupathy & Connor, 1999).

In an attempt to cast light upon the mechanisms underlying this intermediate level of form processing, several studies have investigated human performance for two lines forming an angle (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996; Regan, Gray, & Hamstra, 1996; Snippe & Koenderink, 1994). One specific issue addressed by these studies is whether or not specialized mechanisms exist to compute angles. The notion of special "angle detectors" would be supported if the discrimination of an angle was more accurate than that predicted by the discrimination of the two lines that comprise the angle, and experimental evidence has been provided in support of their existence (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996; Regan et al., 1996). However, this result has not been replicated in all studies (Snippe & Koenderink, 1994).

One implicit assumption in these studies on angle discrimination has been that performance is driven by the stimulus in the local

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region of the angle but is independent of the overall geometry of the shape that contains the angle (e.g. the two remaining angles and/or the third side of a triangle). It has recently been shown that this is not the case and, instead, that discrimination is strongly dependent on the shape that contains the angle (Kennedy, Orbach, & Loffler, 2006). Angles that form part of isosceles triangles can be judged up to three times more accurately than those forming part of scalene triangles. For scalene shapes, performance can be predicted by the sensitivity of orientation discrimination. Evidence for special angle detectors is therefore only seen when an angle is bounded by two lines of equal length (i.e. part of an isosceles triangle). These observations can reconcile the differences between earlier investigations (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996; Regan et al., 1996; Snippe & Koenderink, 1994): the overall pattern shape appears to determine whether or not angle discrimination is better than orientation discrimination (Kennedy et al., 2006).

The conclusion from the study of Kennedy et al. (2006) is that the *discriminability* of angles is intrinsically linked to the overall shape of the triangle that contains the angle. This begs an interesting question: are angles *perceived* differently when they are part of different shapes? To our knowledge, it has not been investigated whether different triangular shapes cause a systematic bias in angle judgments, i.e. whether angles contained in isosceles triangles are always perceived larger or smaller than those contained in scalene triangles.

2. Methods

2.1. Stimuli

Throughout this paper, we term the angle to be judged the "apex angle". This angle in all experiments was part of a triangle. The contrast cross-section profile of its sides was given by the following exponential function (Loffler & Orbach, 2001):

$$f(x,y) = C \cdot e^{-\left(\frac{x}{\sigma_x}\right)^{N_x}} \cdot e^{-\left(\frac{y}{\sigma_y}\right)^{N_y}} \tag{1}$$

Note that Eq. (1) is for a vertical line; other orientations were produced by simple coordinate transformation. Using a higher order exponential function avoids pixelation artifacts (anti-aliasing) for orientations other than horizontal and vertical (Loffler & Orbach, 2001). The space constants, σ_x and σ_y, were chosen to give sides a width of 0.08° and a specified length (see below). The exponents (N_x, N_y) were assigned values of 6 and 30 to give line edges and tips an equally smooth appearance. The contrast (C) of the two lines forming the apex angle was set to +98% and the contrast of the opposite line to −98%, to make it obvious which of the three angles of the triangle had to be judged.

An important objective in the stimulus design was to remove any cue to the task other than the angle (for a more extensive discussion see Kennedy et al., 2006; Regan et al., 1996). Within each block of trials, all triangles had the same initial overall orientation (defined as the angle bisector), which was either vertical (90°), horizontal (180°) or oblique (45°). However, a random amount of orientation "jitter" (up to ±10°) was added to each triangle, in order to eliminate a change in orientation of either of the two lines defining the angle as a reliable cue. The lengths of the two sides defining the apex angle were varied randomly in each triangle (by up to ±20% of the mean length) to remove a change in the length of the side of the triangle opposite the apex angle as a potential cue. All randomizations were made using uniform distributions.

Two types of triangular shape were used in these experiments: isosceles and scalene (Fig. 1). In isosceles triangles, the two sides defining the apex angle (l_1 and l_2) were always of the same length (ratio $l_1/l_2 = 1.0$), although the absolute length was randomized:

$$\begin{aligned} l_1 &= l_{\text{mean}} \pm (\text{rand} \cdot 0.2 \cdot l_{\text{mean}}) \\ l_2 &= l_1 \end{aligned} \tag{2}$$

Here, and elsewhere, l_{mean} is the average length of the two sides defining the angle, and 'rand' is a random number from a uniform distribution [0–1].

In scalene triangles, the two sides defining the angle (l_1 and l_2) always had different lengths. Different scalene shapes were generated by manipulating the ratio of the two sides (l_1/l_2):

$$\begin{aligned} l_{\text{initial}} &= l_{\text{mean}} \pm (\text{rand} \cdot 0.2 \cdot l_{\text{mean}}) \\ l_1 &= l_{\text{initial}} \cdot \left(1 + \frac{k}{2}\right) \\ l_2 &= l_{\text{initial}} \cdot \left(1 - \frac{k}{2}\right) \end{aligned} \tag{3}$$

As for the isosceles triangles, the absolute side lengths (l_{initial}) were randomized across trials. "k" is a constant used to vary the ratio of the length of the sides. In different experimental conditions, different values of "k" (0.5, 1.0, 1.4) were used to create scalene triangles with side length ratios of 1.7:1, 3.0:1, and 5.7:1, respectively (Fig. 1).

It has been reported that angle discrimination thresholds depend on the reference angle, with performance best for angles close to 0°, 90° and 180° (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996), although this has not been replicated in other studies (Regan et al., 1996). In order to carry out a general investigation into the bias caused by different triangular shapes, experiments were carried out using several reference angles (30°, 50°, 60°, 90° and 120°).

In a further experiment, we wanted to investigate the effect of overall triangle size on the illusion. This was achieved by changing the population mean side length of the two lines bounding the angle (l_{mean} in Eqs. (2) and (3)). Mean side lengths were 0.875°, 1.75° or 7.0°.

The area enclosed by an angle (stimulus size) may influence its perceived size (Wenderoth & Johnson, 1984). The design of the isosceles and scalene triangles described above produces isosceles triangles that, on average, have a larger area than scalene triangles. From the general formula for the area within a triangle, where one of its angles is given by α with bounding sides l_1 and l_2,

$$A = \frac{1}{2} l_1 l_2 \ \sin \alpha \tag{4}$$

it can be seen that for triangles sharing identical apex angles (e.g. $\alpha = 60°$) and identical "random" length variations (e.g. set rand = 0 in Eqs. (2) and (3)), a scalene triangle has an area which is smaller by a factor of

$$1 - \frac{k^2}{4} \tag{5}$$

compared to that of an isosceles triangle. In order to determine whether or not this difference in area has an effect on the perceived angular magnitude, an additional experiment was carried out, where the two triangles shown in each trial always had identical areas. This was achieved by using the same random number ('rand') for the side length of each pair of triangles and multiplying the average side length (l_{initial} in Eq. (3)) of the scalene shape by a factor of:

$$\sqrt{\frac{1}{1 - \frac{k^2}{4}}} \tag{6}$$

In a final experiment, triangles were not defined by lines but, instead, by a dot at each corner point. Each dot had a circularly symmetric D4 luminance profile, with a peak spatial frequency of 8 cpd (Kennedy et al., 2006).

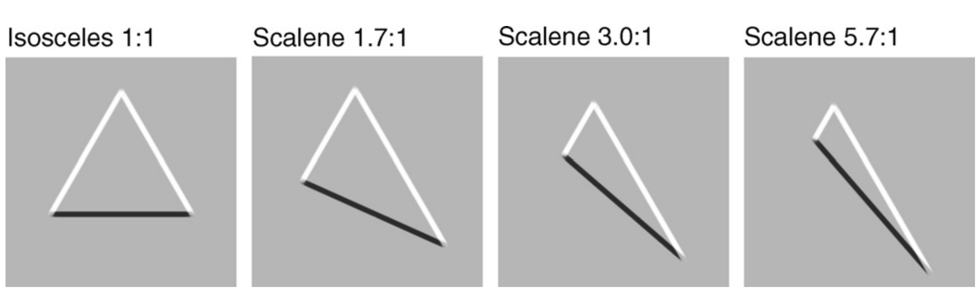

Fig. 1. Examples of triangles used in the experiments, defined by the ratio of the two sides bounding the angle (the two white lines). Triangles were either isosceles ($l_1/l_2 = 1.0$, left) or scalene ($l_1/l_2 = 1.7{:}1$, 3.0:1 and 5.7:1). The four examples share the same apex angle size (the upper angle, 60°) and the same orientation (upright = 90°), which was defined by the apex angle bisector. The angle to be judged was made explicit by assigning the same contrast ('white') to the two lines bounding the angle, which differed from the contrast ('black') of the side opposite the apex.

2.2. Procedure

The screen background was set to mid-grey. A fixation mark, consisting of a small dark circle, appeared on the screen prior to, but not during, each trial and subjects were encouraged to maintain fixation. The method of constant stimuli was employed in a temporal two-alternative forced choice paradigm. One scalene and one isosceles triangle were presented on each trial and the observer indicated, by a key press, the interval in which the triangle had the more obtuse apex angle. Triangles were positioned so that their centres of gravity were at the centre of the screen, with a small amount of positional jitter. The time between pressing a key and the onset of the first stimulus was 200 ms. Each stimulus was presented for 200 ms. To minimize the effects of neural persistence, all stimuli were followed immediately by a mask for 200 ms. The mask consisted of randomly positioned and oriented white lines generated in the same way as the lines defining the triangle.

Two experimental conditions were combined in each experimental run. In one condition, the reference triangles were isosceles and the test triangles scalene, and in the other, the references were scalene and the tests isosceles. Trials from the two conditions were randomly interleaved. This design was used to prevent subjects from learning which triangle shape was the reference and which the test. Hence, within each experimental run, each shape was presented with different angular magnitudes (increments and decrements relative to the angular magnitude tested) and, therefore, both the scalene and the isosceles triangles could contain angles that were more as well as less obtuse than the reference.

For each of these two conditions, angle discrimination performance was measured for seven angular differences between reference and test angle. Based on the results of preliminary experiments, these increments were not distributed symmetrically around the reference angle but were offset by 10°. This offset was made positive when the reference triangles were isosceles and negative when the reference triangles were scalene. By taking into account an expected bias, this strategy allowed us to maximize the likelihood that we were measuring around the point of subjective equality. Despite this offset, due to the fact that scalene and isosceles test angles were interleaved, the average of all test angles was identical to the reference angle, i.e. average test angle was not offset with respect to the reference angle. This minimizes the risk of introducing a methodological response bias away from the reference angle, i.e. a range effect (Poulton, 1979). Different increments were presented randomly in different trials; the order of presentation within trials (i.e. reference angle in the first or second interval) was also random. Each of the seven increments was presented 20 times in each trial sequence, giving a total of 280 trials per experimental run.

The resulting data from each trial sequence (isosceles or scalene as reference) were fitted with a Quick function (Quick, 1974), using a maximum likelihood procedure. A point of subjective equality (PSE or 50% correct point) was determined from each psychometric function. The amount of bias, or magnitude of the illusion, was defined as the shift of the PSE (in degrees) from geometric equality. For each condition, each observer carried out at least two experimental runs on different days, and the separate threshold estimates were averaged.

2.3. Observers

Two of the authors (G.K. and G.L., both male), along with four naïve observers (2 male and 2 female), participated in the first experiment, which was designed to confirm the presence of the main effect (Section 3.1). In subsequent experiments, designed to investigate the influence of various stimulus parameters (Sections 3.2 and 3.4), the number of observers was smaller but always included at least one naïve observer. All observers had normal or 'corrected-to-normal' vision and viewing was always binocular. No feedback was given to the observers as to their performance.

2.4. Apparatus

Stimuli were presented on a LaCie "electron22blue" high-resolution monitor controlled by an Apple PowerMac G4 computer. The frame refresh rate of the monitor was set to 85 Hz and the spatial resolution to 1024 × 768 pixels. The software lookup table was defined to maximize contrast linearity using 151 equally spaced grey levels. Pattern luminance was modulated about a mean of 61 cd/m^2. Subjects viewed the stimuli under dim room illumination and a chin and forehead rest was used to maintain a constant viewing distance of 120 cm. At this distance each pixel subtended 0.018°. To avoid reference cues, the monitor frame was covered with a white cardboard mask with a circular aperture subtending 12° in diameter. Individual patterns were calculated in MATLAB prior to the experiments. The patterns were displayed using custom-written Pascal code within the CodeWarrior environment. Experimental programs included routines from Pelli's VideoToolbox (Pelli, 1997).

3. Results

3.1. The basic illusion

In pilot experiments under informal inspection, when asked to compare the size of two identical angles presented side-by-side (see Fig. 2), the majority (79%) of observers ($N = 19$) reported an angle in a scalene triangle to be less obtuse than in an isosceles triangle. Even those who did not report seeing a difference with the initial presentation matched a physically smaller angle in an isosceles triangle with a larger angle in a scalene triangle, when presented with several triangle pairs and asked to pick a match. There are several reasons why some observers might not see the illusion on unlimited and unrestricted observation. Observers can scan the two stimuli making eye-movements and the signal from the extra-ocular muscles might be used to counter-act the perceptual illusion. It has been shown that observers are remarkably good at making judgements of angular size using only saccadic eye-movements (Hayhoe, Lachter, & Feldman, 1991). We employed a short presentation time in the formal experiments to rule out this strategy. Another strategy to determine if two angles are the same or not is to compare absolute orientations, e.g. comparing the orientations of corresponding lines bounding the two angles. If the overall triangle orientation is the same (as is the case in Fig. 2), the orientations match and it follows logically that the two angles must be the same. Note that the potential to use this strategy was also removed in the formal experiments, where the orientation of the triangles to be compared was randomized.

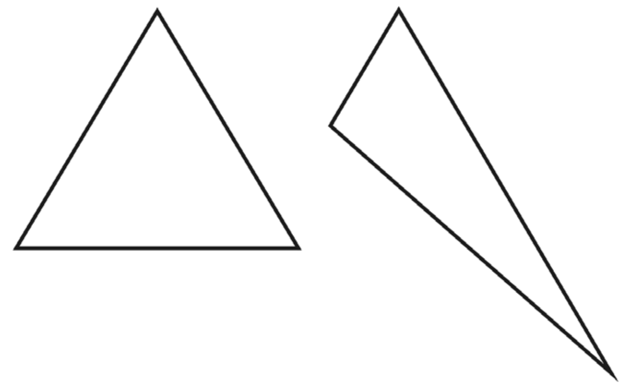

Fig. 2. The angle illusion presented in this paper. When asked to compare the upper angles in the two triangles, observers typically report that the isosceles triangle (left) had the larger angle compared to the scalene shape (right) although both angles are identical (60°). This suggests that the perceived angular size depends on the shape of the triangle that contains the angle.

To confirm and quantify this observation, a set of formal psychophysical experiments was run. Offsets (shifts of PSE from geometric equality) are shown in Fig. 3 for a reference angle of 60°, comparing isosceles and scalene triangles with a side length ratio of 3.0:1. The mean side length of the triangles was 1.75°. The left hand plot shows the results from each of the two psychometric functions, one using isosceles triangles as the reference (white bar) and the other using scalene (black bar). For an isosceles reference triangle, the angle in the scalene test triangle had to be approximately 12.5° larger to be judged as the same. When the scalene triangle was the reference, angles in isosceles triangles were judged the same when they were about 15° smaller. The difference between these two magnitudes was not statistically significant ($p = .28$), so the data for the two reference types were averaged (unsigned) and are shown on the right. These data can be understood as the amount by which an angle in a scalene triangle has to be larger than an angle in an isosceles triangle in order for the two to be perceived as the same. All further discussion relates to averaged values for the two reference shapes.

The shift of PSE in this condition (14°) is considerable. Given that the reference angle in this experiment was 60°, this represents a misperception of almost 25% of the angular size.

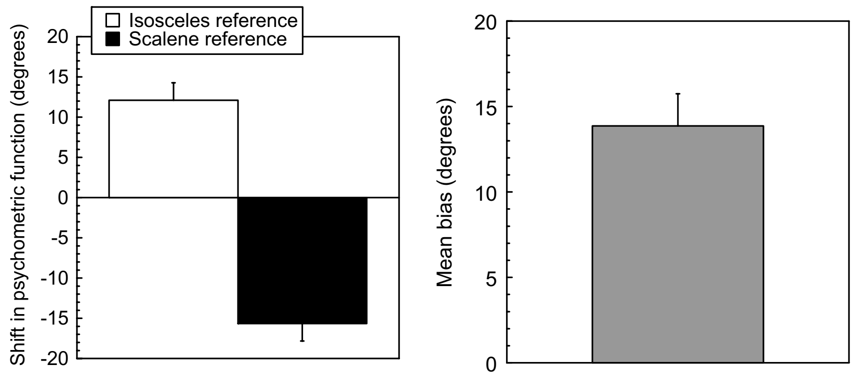

Fig. 3. Results for the basic illusion. Shift of PSE (from geometric equality) when comparing isosceles with scalene triangles (side length ratio of 3.0:1, reference angle of 60°, average side length of 1.75° and orientation randomized around the vertical). Data are averaged across six observers. Error bars here and elsewhere are standard errors of the mean. The left plot shows results determined from the two separate experimental conditions where either the isosceles (white bar, positive shift) or the scalene triangle (black bar, negative shift) was used as the reference. Given the similarity of the absolute values, the data were averaged and the magnitude of the shift (unsigned average) is shown on the right. Observers perceive angles that are part of scalene triangles as substantially (approximately 14°) smaller than those that are contained in isosceles triangles.

3.2. Effect of triangle shape

In order to investigate the effect of shape, a second experiment was carried out using scalene triangles with two additional side length ratios, one smaller (1.7:1) and one larger (5.7:1) than that used in the previous experiment (3.0:1). Mean offsets of the PSE for these two side length ratios, averaged across four observers, are shown in Fig. 4A. For comparison, these observers' data for a side length ratio of 3.0:1, obtained in the previous experiment, are also shown. It is clear that when the side length ratio is 1.7:1, the magnitude of the illusion is reduced. On the other hand, when the side length ratio is 5.7:1, offsets are very similar to that seen for a ratio of 3.0:1. Statistical analysis using an ANOVA confirms that there is an effect of changing the side length ratio ($F_{2,9} = 29.53$, $p < .01$). Post-hoc tests (Fisher's PLSD) were carried out and confirm that the data for a side length ratio of 1.7:1 is significantly different to that for the other two ratios ($p < .01$ in both cases). There is no significant difference between ratios of 3.0:1 and 5.7:1 ($p = .30$). This suggests that an asymptotic level of illusion has been reached for a ratio of about 3:1.

One advantage of plotting psychometric functions is that, in addition to determining PSEs, one can also easily estimate sensitivities, or angle discrimination thresholds. These can then be used to determine whether any stimulus manipulation that appears to influence the PSE (side length ratio in this case) also causes a change in sensitivity. For each observer and each side ratio we determined the threshold as half the distance between the 25% and 75% points on the psychometric function. For ratios of 1.7:1, 3.0:1 and 5.7:1, mean thresholds were 4.77°, 5.15° and 5.97°, respectively. Statistical analysis shows no effect of shape on these thresholds [ANOVA ($F_{2,9} = 1.58$, $p = .26$)]. These threshold values correspond to the standard concept of "just noticeable difference" (JND) and represent the amount by which two angles have to differ so that observers can reliably discriminate them. Thus, it appears that, although increasing the side length ratio causes a significant increase in the shift of PSE, it does not cause a significant increase in angle thresholds. When expressed as multiples of threshold, PSE offsets for the three ratios are equivalent to 1.45, 2.94 and 2.79 JNDs, respectively.

3.3. Effect of reference angle

The results from the first two experiments show the presence of an illusion for a reference angle of 60°. In these cases, when the reference triangles were "isosceles", they were actually equilateral. It may be that this special geometric shape is encoded differently,

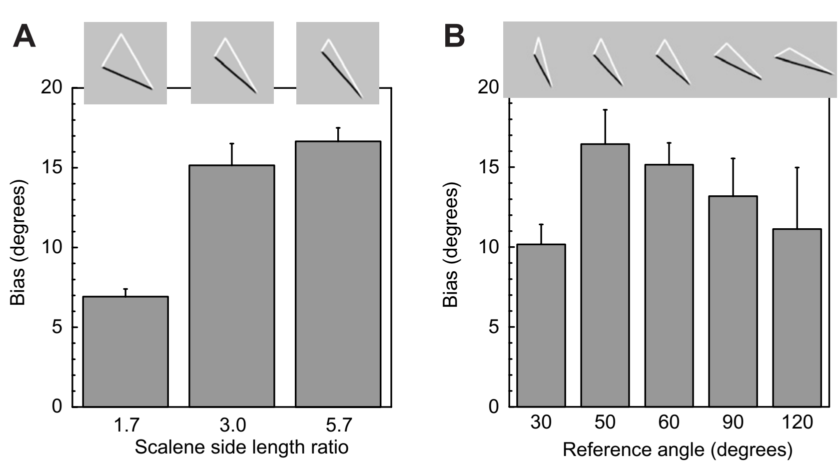

Fig. 4. Mean data ($N = 4$) for three scalene side length ratios (A) and five reference angles (B). In A, when the side length ratio is reduced from 3.0:1 to 1.7:1, the shift of the PSE is reduced ($p < .01$). However, when the ratio is increased to 5.7:1, shifts are not significantly larger than for a ratio of 3.0:1 ($p = .30$). In B, significant effects ($p < .0001$) exist for all five reference angles (30°, 50°, 60°, 90° and 120°). However, statistical analysis shows no main effect of reference angle ($p = .34$). The side length ratio was 3.0:1 and the mean side length was 1.75°.

and that the illusion only occurs when comparing equilateral with non-equilateral triangles. It has also been reported that the accuracy of angle discrimination can depend on the magnitude of the angle to be judged (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996). To investigate the generality of the effect, a third experiment was carried out using four other reference angles (30°, 50°, 90° and 120°).

The magnitude of the effect for these four angles is shown in Fig. 4B. It is clear that a shift of PSE is present for all reference angles tested. We assessed the presence of the illusion for each reference angle statistically by a one-sample analysis with a hypothesized mean of zero, which would result if angles in isosceles and scalene triangles were judged the same. The analysis underlines that the match is significantly different from zero for each reference angle ($p < .0001$).

Thus, whether acute, obtuse or 90°, an angle contained in a scalene triangle is perceived smaller than an angle forming part of an isosceles triangle. While the magnitude of the bias is clearly not identical for different reference angles, statistically there is no main effect [ANOVA ($F_{4,15} = 1.22$, $p = .34$)].

As in the previous experiment, for each reference angle we can also provide a measure of observer sensitivity. For reference angles of 30°, 50°, 60°, 90° and 120°, mean angle discrimination thresholds were 4.58°, 4.75°, 5.15°, 6.51° and 9.52°, respectively. Thus, angle thresholds appear to follow the same general pattern found in some previous studies, namely that thresholds increase with reference angle (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996). We do not, however, see lower thresholds for right angles that have been reported previously (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996). Expressed as multiples of discrimination thresholds, PSE offsets are equivalent to 2.2, 3.5, 2.9, 2.0 and 1.2 JNDs, respectively.

3.4. Effect of other stimulus parameters

Five additional parameters were investigated: stimulus orientation, stimulus size, stimulus area, stimulus closure and stimulus outline. For orientation, the magnitude of several visual illusions (e.g. Zöllner and Poggendorff illusions) has been shown to depend on the overall orientation of the figure, i.e. whether the figure is oriented obliquely or along one of the principal meridians (Robinson, 1972). For stimulus size, the perceived magnitude of an angle has been shown to increase with the overall size of the stimulus (Werkhoven & Koenderink, 1993), possibly related to the area enclosed by the stimulus (Wenderoth & Johnson, 1984). For closure, omitting the side opposite the angle might weaken the effect of the overall triangle and therefore reduce the illusion. Finally, it has been suggested that the mechanisms used to compute outlined angles are fundamentally different to those involved when angles are defined by dots at the triangle corners (Heeley & Buchanan-Smith, 1996).

In order to investigate the influence of these factors discussed, we carried out further experiments where we manipulated stimulus orientation (Fig. 5A), stimulus size (Fig. 5B), area enclosed by the stimulus (Fig. 5C), stimulus closure (Fig. 5D), and the method used to define the stimuli (i.e. lines vs. dots, Fig. 5E). None of these factors has an influence on the magnitude of the illusion ($p > .40$ in all cases).

4. Discussion

4.1. A new angle illusion

It has been shown previously that angles forming part of isosceles triangles can be *discriminated* with greater precision than those that form part of scalene triangles (Kennedy et al., 2006). It was not known whether perceived angular size also depends on the shape of a triangle. The results of the first experiment show that observers perceive angles in scalene triangles as substantially smaller (up to 25% of the angular magnitude) than angles in isosceles triangles, confirming a novel geometric visual illusion. This effect is independent of reference angle, stimulus orientation and stimulus size. It is striking that both the precision of angle discrimination and the perceived size of angles are affected by the global shape of the triangle, since one would presume such judgements to be simply based on local information at the meeting point of two lines.

The effect we report is somewhat paradoxical. If the apex angle of a triangle is increased in size, the length of the triangle base also increases. It has been suggested that observers could use this cue when making judgments of angles (Regan et al., 1996). However, despite the fact that the scalene triangles in these experiments have, on average, a longer base than isosceles triangles, their apex angles are perceived as smaller.

Among the possible factors contributing to the illusion, differences in the areas enclosed by triangles can be ruled out. It has been reported previously that angles are judged larger as the area they enclose is increased (Wenderoth & Johnson, 1984; Werkhoven & Koenderink, 1993). However, the illusion remains undiminished when the areas enclosed by isosceles and scalene triangles are matched (Fig. 5C).

Given the importance of the global stimulus geometry in this illusion, the possibility was examined that the illusion might fundamentally depend on the stimulus being a closed shape rather than two intersecting lines. It might be argued that the former configuration strengthens a global interpretation while the latter might de-emphasize it. The results of a further experiment show that the illusion persists even when only the two sides defining the angle are shown (Fig. 5D). In other words, an angle defined by two lines of different lengths is perceived smaller than an angle defined by two lines of equal length. The presence of the third side is not critical for the illusion to occur.

4.2. Relation to other illusions

It is well-known that the perceived orientation of a line can be influenced by the presence of a second intersecting or abutting line (Blakemore, Carpenter, & Georgeson, 1970; Bouma & Andriessen, 1970; Greene & Levinson, 1994; Wenderoth & Johnson, 1984). This phenomenon is known as "tilt contrast" and has been used to explain various geometric visual illusions (e.g. Zöllner and Poggendorff illusions). Take, as an explicit example, the Zöllner illusion, where several short, oblique segments intersect long, vertical lines. Each intersection of a vertical and an oblique line features two acute and two obtuse angles. It has been proposed that the short oblique segments induce a shift in the perceived orientation of the long lines in a direction away from the acute angle of their intersection and towards the obtuse angle. This leads to the acute angle being perceived larger than it is and the obtuse angle smaller. Hence, one consequence of tilt contrast is that acute angles can be overestimated in size, and obtuse angles underestimated (e.g. Blakemore et al., 1970; Jastrow, 1892).

The classical over-estimation of acute and/or under-estimation of obtuse angles cannot explain the results of the current experiments. Firstly, the illusion seen with acute and obtuse angles is typically in the region of 1–2° (Blakemore et al., 1970; Nundy, Lotto, Coppola, Shimpi, & Purves, 2000), approximately an order of magnitude smaller than the effect reported here. Secondly, the critical parameter for the current illusion is not an over- or under-estimation of an angle but an over- or under-estimation of an angle that depends on the characteristics (lengths) of the angle's bounding lines.

More relevant to our observation would be any studies of the dependence of tilt contrast on relative line lengths. Robinson

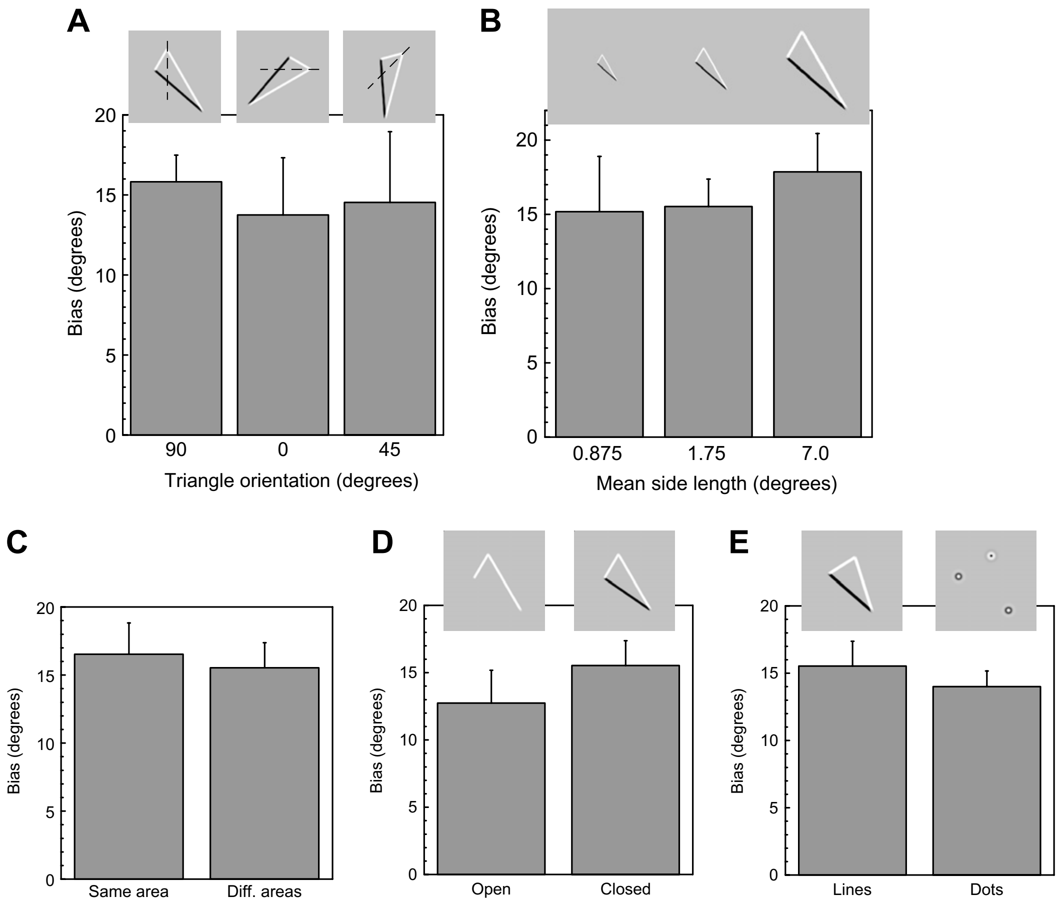

Fig. 5. Mean data for various stimulus manipulations. Experiments were carried out where stimulus orientation (A), stimulus size (B), area enclosed by the stimulus (C), stimulus closure (D), and the method used to define the stimuli (E), was varied. Data in each case are averaged across three observers. None of these parameters was found to have any significant effect on the magnitude of the illusion ($p > .40$ in all cases).

(1972, p. 89), for example, observed that when testing an acute angle made up of a short and long arm, the short arm appeared tilted away from its real position much more than the long arm, as though the short arm was influenced more by the long arm than vice versa. However, it has also been reported that short inducing segments can produce a significant effect on a longer test line (Greene & Levinson, 1994). A disproportionately strong perceptual shift of a short by a long arm would, however, predict the opposite illusion: the angle in a scalene triangle should be more obtuse than that in an isosceles triangle.

Our study demonstrates that the perception of angular size is affected by the context in which an angle is presented. In this respect, it shares some similarities with the well-known Titchener illusion, where a circle surrounded by an array of smaller circles appears larger in size than a circle of the same size surrounded by larger circles. In both illusions, two features that have the same physical properties are perceived differently because of the context within which they are presented. The context in the Titchener illusion is the surrounding circles, in our experiments it is the overall shape of the triangle. The Titchener illusion is often attributed to a size-contrast effect, where an object that is larger than those adjacent to it is assumed to be larger than one that is smaller than its neighbours (Robinson, 1972). Alternatively, it has been suggested that the effect is due to a correction for perceived image-distance (Gregory, 1963). In this explanation, an array of small circles is perceived as being more distant than an array of larger ones, causing its central circle (which is also assumed to be more distant) to be perceptually enlarged compared to a central circle with the same retinal image size in an array of larger circles. It is not obvious how either of these two explanations, size-contrast and correction for image-distance, may account for the effect seen with triangle shapes.

Given the analogy to the Titchener illusion we have also described our effect as an "illusion". A reviewer pointed out that it may be misleading to refer to such perceptual effects as illusions, as this implies that they are inaccurate representations of a fixed "real world" that are created by imperfect visual machinery. Rather, it may be more appropriate to consider the concepts of egocentric (or viewer-centred) representations and allocentric (object-centred) representations. Such a distinction can be made for the Titchener illusion, where visual perception but not visually guided motor actions (i.e. when asked to reach out and grasp a disc in the centre of an array of larger or smaller discs, observers' grip aperture matched the physical rather than the perceived size of the central disc) are affected (Aglioti, DeSouza, & Goodale, 1995). Aglioti et al. (1995) argue that the visual interpretation of scenes is driven by representations in an object-centred coordinate system, in which distortions can occur due to interactions between objects, whereas the judgements required for making skilled motor actions are driven by viewer centred-representations that are not susceptible to such distortions.

4.3. Implications for angle mechanisms

The most straightforward computational model for angle discrimination is an hierarchical one based on the orientations determined for the two bounding lines (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996; Regan et al., 1996). Such models face a key challenge: Angle discrimination can actually be better than that predicted by combining the sensitivities for the determination of the two orientations (Heeley & Buchanan-Smith, 1996; Kennedy et al., 2006; Regan et al., 1996). To account for this, models with two parallel mechanisms for orientation and angle discrimination have been proposed (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996). Both mechanisms include a second stage computation, which combines the outputs of the same first-stage channels (e.g. orientation selective simple cells in V1). The difference between the two is the amount of noise added at the second stage. If this noise is stronger for orientation than for angle measurements, this explains why orientation acuity is poorer than angle acuity (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996).

Our results add a second challenge for models based on orientation: both the precision (Kennedy et al., 2006) and the accuracy (current study) of angle perception depend on triangle shape. Because the orientations of the bounding lines are the same, orientation based models, whatever the nature or amount of noise, would not predict this behaviour. The same argument rules out curvature mechanisms underlying angle perception (Wilson & Richards, 1989) since the information available to the curvature detector in the vicinity of the angle also does not depend on triangle shape.

One obvious difference between isosceles and scalene triangles is that the former exhibit a symmetry that the latter lack. Bilateral mirror symmetry abounds both in nature and man made objects, and symmetry is known to be processed very efficiently by the visual system (Barlow & Reeves, 1979; Wagemans, 1997). One reviewer pointed out that the data shown in Fig. 5E (where the magnitude of the illusion for triangles defined by three dots is shown to be as large as for triangles defined by lines) suggests symmetry as an important factor. In this case, there are no oriented edges and no local angle: the angle must be derived through interpolation between the dots. In other words, the effect remains when observers make a comparison between two dot patterns, one with mirror symmetry and one without. It is obviously impossible to distinguish mirror axis symmetry from the equality of limbs in an isosceles triangle, but one could test the generality of symmetry by using shapes other than triangles. For example, an angle contained in a rhombus or kite (which have mirror symmetry) could be compared with one contained in an irregular quadrilateral (non-symmetric). In any case, even if angles in isosceles triangles were processed with higher precision than those in scalene shapes because of their bilateral symmetry, it is unclear how symmetry could predict the angle illusion reported here.

As an alternative to orientation or symmetry, the visual system might utilise distance or separation judgements. A simple strategy would be to relate angular magnitude to the length of the line opposite the apex angle. However, angle acuity remains unaffected when this cue is unreliable (Kennedy et al., 2006; Regan et al., 1996, present study). Alternatively, one could use the base of the triangle, that is, a line orthogonal to the angle bisector. Such a 'width' measurement would have to be combined with a second measurement (e.g. 'height'), otherwise perceived angular magnitude would scale strongly with triangle size, which is not the case (Werkhoven & Koenderink, 1993).

Combining two linear measurements has been shown to be an attractive candidate to explain the high sensitivity observed in aspect ratio judgements for squares and circles (Morgan, 2005; Regan & Hamstra, 1992; Zanker & Quenzer, 1999). Observers in Regan and Hamstra's (1992) study were able to discriminate be-

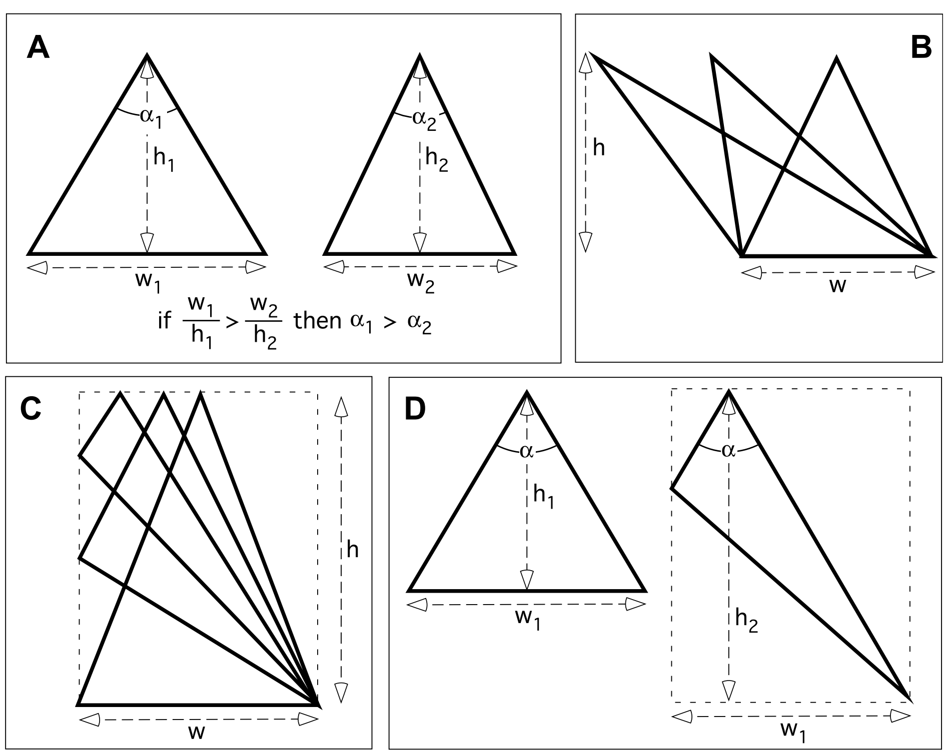

Fig. 6. Aspect-ratio calculations for angles. (A) For isosceles triangles, as the ratio of width (length of the side opposite the apex angle) over height (distance between this line and the apex angle) decreases, so does the angle. The angle can be directly computed from the aspect ratio, independent of triangle size. (B) This, however, only holds for angular magnitudes of isosceles triangles. Using an aspect ratio computation based on the above geometrical definition of heights and widths, the same aspect ratio does not correspond to the same angular magnitude when comparing angles across shapes. Note that in this case the isosceles shape contains the most obtuse angle. (C) Using a different definition of height (in terms of the height of the entire triangle parallel to the angle bisector) and width (width of the entire triangle perpendicular to the angle bisector) gives a better estimate of an angle (C), but still shows a dependence on shape. Note that in this case the isosceles shape contains the least obtuse angle. (D) Comparing the aspect ratios of identical angles embedded in different triangle shapes. Considering height along the angle bisector and width along a line perpendicular to it, a scalene triangle gives a lower aspect ratio than an isosceles triangle, consistent with the illusion reported in this study.

tween a square and a rectangle when the aspect ratios differed by as little as 1.6%. For an isosceles triangle, taking the geometrical definition of 'width' as the length of the side opposite the apex angle and the distance between this line and the apex angle as 'height' (Fig. 6A), this precision translates to a just-noticeable difference of angle magnitude of less than 1° (e.g. 0.73° for an apex angle of 53° where height equals width), which is good enough to capture the highest acuity reported in the literature (Chen and Levi, 1996; Heeley and Buchanan-Smith, 1996; Regan et al., 1996).

Aspect ratios correlate with angular magnitude (Fig. 6A), even when the overall stimulus size is varied, as long as the triangle is isosceles in shape. The situation is more complicated for scalene shapes. For a start, it is less clear what the appropriate height and width for an aspect ratio judgment for a scalene triangle is. Using an aspect ratio computation based on the above geometrical definition of heights and widths would result in vastly different angles being judged the same (Fig. 6B). Using a different definition of height (in terms of the height of the entire triangle parallel to the angle bisector) and width (width of the entire triangle perpendicular to the angle bisector) gives a better estimate of an angle (Fig 6C), but still shows a dependence on shape. In either case, when comparing angles across shapes (i.e. scalene versus scalene or scalene versus isosceles), the same aspect ratio does not correspond to the same angular magnitude. Therefore, using aspect ratios when comparing angles in different triangle shapes will inevitably yield lower sensitivity compared to when triangles are always isosceles, in agreement with experimental results (Kennedy et al., 2006).

We propose that the visual system has access to at least two mechanisms for determining angular magnitude and discriminating angles. One computes the orientations of the two bounding lines and calculates the angle as the difference between them. The other utilises the aspect ratio of the triangle. The overall response is determined by the more sensitive mechanism. In the case of isosceles shapes, this is the aspect ratio. In the case of scalene shapes, it is either the (less reliable than for isosceles triangles) aspect ratio or the orientation computation. The high sensitivity of aspect ratio judgements can explain why angle discrimination can be better than predicted by orientation channels (Chen and Levi, 1996; Heeley and Buchanan-Smith, 1996; Regan et al., 1996), if the angle is part of an isosceles triangle (Kennedy et al., 2006). In contrast, the comparatively low angle acuity for scalene shapes is predicted by the precision of two independent orientation judgements (Kennedy et al., 2006).

The aspect ratio computation is also a potential candidate to explain why the same angular magnitude appears different in an isosceles compared to a scalene triangle. One of the two aspect ratio computations outlined above leads to errors similar to those observed in the present study, while the other would predict the opposite illusion. If height was calculated along the angle bisector and width along a line perpendicular to it (Fig. 6D), the resulting aspect ratios for the same angle depend on the shape of the triangle. In this case, the aspect ratio in a scalene triangle is smaller than in an isosceles triangle, correctly predicting the illusion that identical angles in scalene triangles are perceived as less obtuse than in isosceles shapes. This aspect ratio judgement is centred on the angle bisector, unlike the geometric definition where the height does not relate to its angle bisector (Fig. 6B). It is interesting to note that, unlike the angle itself, the orientation of the angle bisector can be discriminated with similarly high precision and accuracy for both scalene and isosceles triangles (data not shown).

Our results indicate that the global aspects of a stimulus are determining factors when encoding basic shape properties, such as angles. The utilisation of aspect ratio measurements when judging angles is a novel hypothesis that can be tested by, e.g. manipulating the sides of scalene triangles or introducing more complex shapes (e.g. quadrilaterals). It suggests that for a strikingly 'low-level' judgement the visual system employs sophisticated global mechanisms.

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